ZALLEN ENGINEERING
FORENSIC ENGINEERING IN CONSTRUCTION®
ONLINE EDITION
No. 15 March 2008

# Beam and Arch Action

## A Structural Primer for Non-Engineers

by Rubin M. Zallen, P.E

**Figure 1 shows a simple** beam with a distributed vertical load. In the figure, the *centroid* is the geometric center of the cross-section of the beam, and the vertical reactions are the resisting forces from the vertical supports.

If one cuts the beam at any location along its length, such as at Section A-A, removes one side (in this case the right side), and replaces the part that has been removed with the internal forces or stresses, one obtains what structural engineers call a *free body,* as shown in Figure 2. The force V is called *shear*; the sum of the shear and the vertical load on the free body must equal the left vertical reaction for equilibrium. The circular arrow, M, is a rotational force called *moment*, and since this force on the beam bends the beam, it is also called *bending moment.* A moment is the rotational effect of a force about a particular point, and is equal to the force times the perpendicular distance from the line of action of the force to the point. For equilibrium in Figure 2, the moment of the left vertical reaction minus the moment of the distributed vertical load, both about the centroid at Section A-A, must be equal to M.

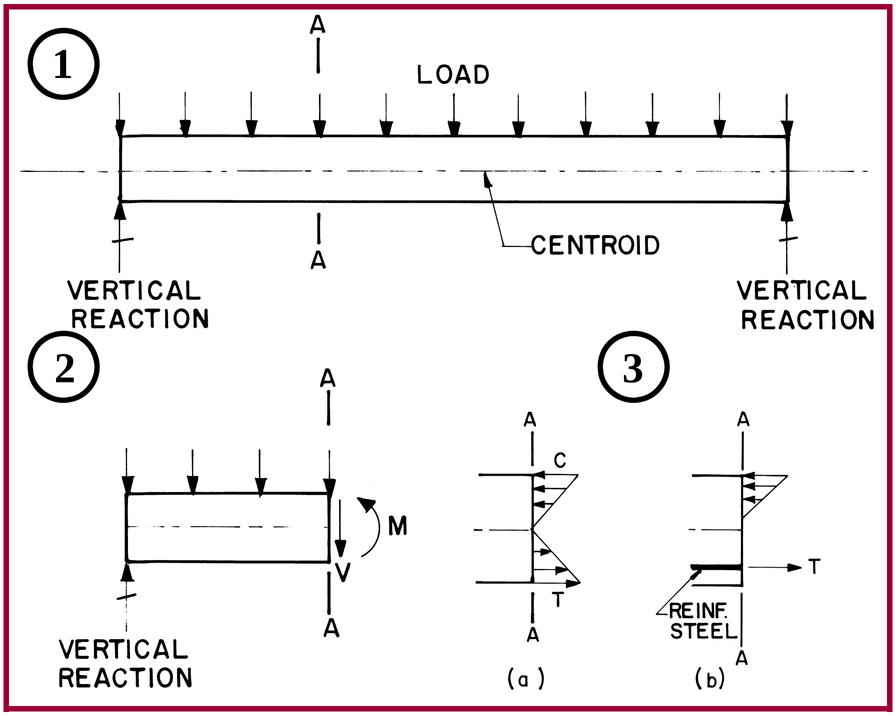


Figure 3a shows the stress distribution due to the moment M of Figure 2. There is a maximum compressive stress, C, at the top, zero stress at the centroid, and a maximum tensile stress, T, at the bottom. The net total stress on the cross-section parallel to the axis of the beam is equal to zero; i.e. there is no net axial force, just the rotational effect. The stress distribution in Figure 3a applies to beams made of materials such as steel or wood which have both tensile and compressive strength. The stress distribution for reinforced concrete is shown in Figure 3b; the concrete has a low tensile strength which is ignored, so that all of the tensile stress is assumed to be resisted by the reinforcing steel.

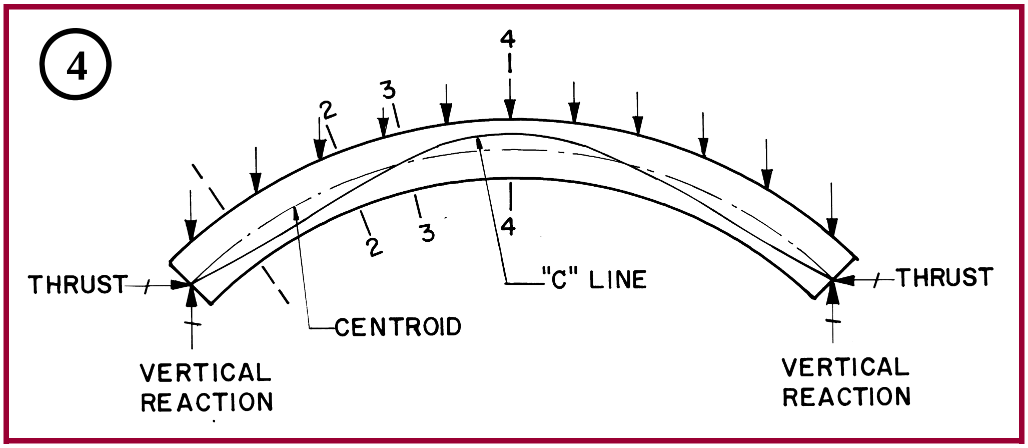
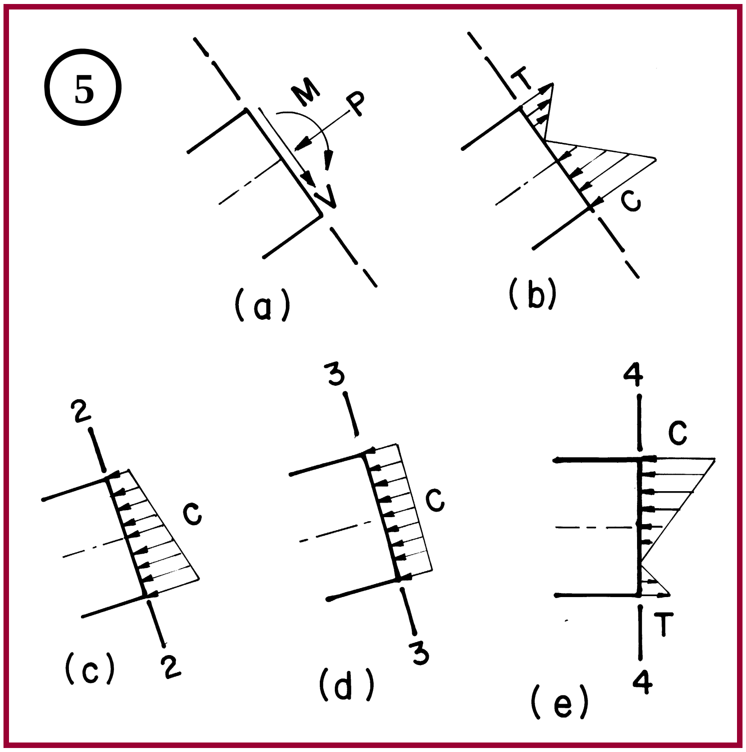
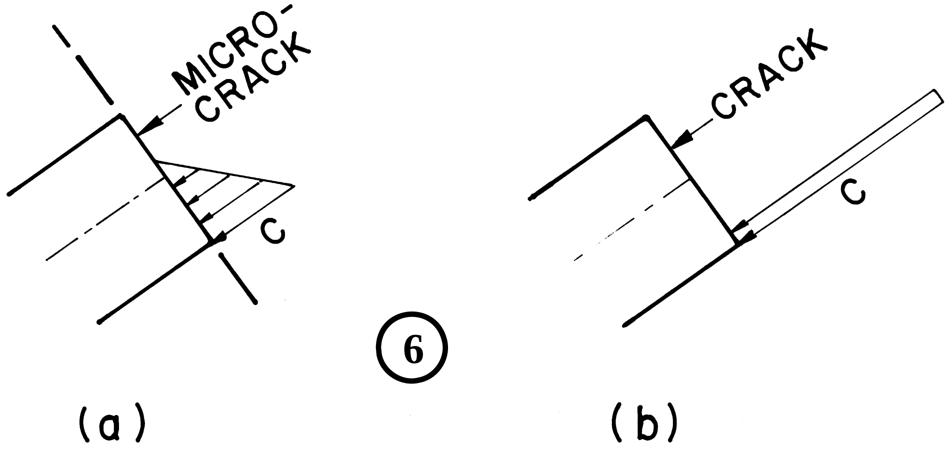


Figure 4 shows an arch, loaded with a distributed vertical load. Distinct from a beam, an arch has a *rise* (the crown is higher than the supports) and a lateral support at each end called a *thrust.* (Without the thrust, the structure would be a curved beam). Figure 5a shows the internal forces at Section 1-1 of the arch, which are typical throughout the arch. The same as for the beam, there is a bending moment, M and a shear, V; however, there is also a concentric axial compression, P, due to the combined effect of the thrust and vertical reaction at each support. The resultant of the bending moment and the concentric axial compression at a cross-section is a single axial compressive force, eccentric from the centroidal axis of the arch (except when the moment is zero). The trace of this compressive force is the *C Line* shown on Figure 4. Figures 5b, 5c, 5d, and 5e show the stress distribution due to the combined effects of the bending moment and the concentric axial compression at cross-sections 1-1 through 4-4, respectively. The axial compression has a dominating effect: at Sections 2-2 and 3-3, there is only compressive stress; and at Sections 1-1 and 4-4, the maximum tensile stress is much lower than the maximum compressive stress.



Figures 5b and 5e only apply to arches made of materials with both tensile and compressive strength, and thus do not apply to masonry or unreinforced concrete. Although masonry and unreinforced concrete have some tensile strength, the tensile strength is very low when compared to the compressive strength, and since it has little effect on the behavior of masonry or unreinforced concrete arches, it is ignored. Figure 6a shows the stress distribution at Section 1-1 of the arch in Figure 4 for a masonry arch. The stress distribution at Section 4-4 is similar but reversed. The stress on the section is only compressive, and where there is no stress, there is a micro-crack. As cracking progresses, there is some redistribution of stresses along the arch. As long as the C Line is within the depth of the arch, a masonry arch will be stable. Under some circumstances, the cracking at a cross-section will be severe, leading effec-



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tively to the formation of a “hinge” (free rotation at the section) and a major redistribution of internal forces throughout the length of the arch.

Figure 6b shows the condition where the cracking at a section is severe. There will be some local crushing of the masonry at the compression face of the arch. The C Line will remain just inside of the compression face so that the compressive stress times the area over which it acts will be less than the compressive strength of the masonry. There will also be sufficient hinging at the cross-section and redistribution of stress throughout the arch to maintain equilibrium under this condition.

There is a limit to the number of hinges that an arch can sustain, beyond which the arch becomes unstable and collapses. The supports for the arch shown in Figure 4 are assumed to be hinged (i.e. there is no moment resistance at the supports). If the arch is subjected to symmetrical distributed loading of ever increasing intensity, a hinge will form at Section 4-4 (at the crown). If the load is further increased a hinge will eventually form between the crown of the arch and one of the supports; the arch would then become unstable and collapse.

The principal causes for collapse of masonry or unreinforced concrete arches, which is a result of the hinging just described, is inadequate depth of the arch cross-section and non-symmetrical loading.

Masonry and unreinforced concrete arches need special analysis techniques that account for “hinging” and the location of the C Line at a “hinge.” The author has developed a limit states design method for this purpose.

Forensic Engineering in Construction® is published by Zallen Engineering, 1101 Worcester Road, Framingham, MA 01701. Comments are welcome. Please direct comments to Rubin M. Zallen, P.E., by telephone at 508-875-1360 or by e-mail at rmzallen@zallenegineering.com.